Goals of Discrete Math

* Mathematical Reasoning
  + Read, conclude, prove the problem. It’s about the science and out of proof methods.
* Combinatorial Analysis
  + Counting or enumerating objects by not just applying formulas.
* Discrete Structure
* Algorithms Thinking
  + Certain classes of problems are sorted by algorithms an then we are implementing the specification of algorithm and verification
* Modelling and applications
  + Problems in other areas like biology, geology, chemistry, marketing and business can model and get by using discrete math techniques.

Chapter 1

Logic and Proofs

Propositional logic

Definition: A proposition is a declarative sentence (a sentence that declared a fact) that is either true or false, but not both

The conventional letters we use for propositional are T & F

Compound Propositions

Definition 2: Let P be a proposition. The negation of p, denoted by Xp, is the statement “it is not the case that p”

The proposition Xp is read “ not p”

The true value of the negation of p and Xp is the opposite of the true value of p

|  |  |
| --- | --- |
| P | Xp |
| T | F |
| F | T |

Definition 3: let P and Q be proposition, the conjunction of P and Q, denoted by P^Q, is the proposition “P and Q”. The truth value of P^Q is true when both P and Q are true

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| P | Q | P^Q | PvQ | P exclusive or Q |
| T | T | T | T | F |
| T | F | F | T | T |
| F | T | F | T | T |
| F | F | F | F | F |

Definition 4: Let P and Q be propositions, the disjunction of P and Q, denoted by PvQ, is the Proposition P or Q. PvQ is false when both P and Q are false.

Definition 5: Let P ad Q be proposition. The exclusion or of P and Q, denoted by P exclusive or Q.

P exclusive or Q is the proposition which is true when exactly one of P and Q is true.

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Disjunctive Normal Form

* A proposition constructed by taking the OR of ANDs

Theorem – Any compound proposition is logically equivalent to one in DNF

(P Λ ¬Q Λ R) V (P Λ ¬Q Λ R)

<------------->

^ is a clause

(P V Q) 🡪 R

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| P | Q | R | (P V Q) 🡪 R | |
| T | T | T | T | T |
| T | T | F | T | F |
| T | F | T | T | T |
| T | F | F | T | F |
| F | T | T | T | T |
| F | T | F | T | F |
| F | F | T | F | T |
| F | F | F | F | T |

Definition: A set of operators is functionally complete if any proposition has an equivalent proposition that uses only the operators in the set.

Predicates and Quantifiers

P (x): x > 3

P (4) = T

P (2) = F

Q (X,Y): X > Y

Q(3,2): T

Q(2,3): F

∀ = for all

∃ = there exists

∀x∀y Q(X,Y)

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+-Nested Quantifiers